Chapter 5

Jackson: 5.3, 5.6, 5.10, 5.13, 5.14, 5.18(a),(c), 5.19(a), 5.21, 5.22, 5.26, 5.27

A5.1 A steady current $I$ flows down an infinite straight wire of radius $R$. Find the magnetic field $\mathbf{B}$ and vector potential $\mathbf{A}$ inside and outside the wire.

A5.2 Two long coaxial solenoids each carries current $I$ in the same direction, as shown in Fig. 1. The inner solenoid (radius $a$) has $n_1$ turns per unit length, the outer (radius $b$) has $n_2$. Draw Ampere’s loops for the vector potential $\mathbf{A}$ and find their values in each of the three regions: (a) $s < a$, (b) $a < s < b$, (c) $s > b$.

A5.3 Two very long coaxial cylindrical tubes (with radius $a$ and $b$ respectively) are separated by linear insulating material of magnetic susceptibility $\chi_m$. A current $I$ flows down the inner conductor and returns along the outer one (Fig. 2); in each case the current distributes itself uniformly over the surface. Find: a) the magnetic field $\mathbf{B}$ at $a < s < b$ and $s > b$; b) the surface bound current $\mathbf{K}_M$ at $s = a$ and $s = b$; c) the volume bound current $\mathbf{J}_M$.

A5.4 A plane loop of irregular shape is situated so that part of it is in a uniform magnetic filed, which points into the page in the region as shown in Fig. 3. A mass $m$ is hung on the loop vertically. The distance between points $a$ and $b$ is $s$. For what current (direction and magnitude), in the loop, would the magnetic force upward exactly balance the gravity downward?
A5.5  
(a) Find the magnetic field $\mathbf{B}$ a distance $z$ above the center of a circular loop of radius $R$, which carries a steady current $I$ as shown in Fig. 4(a).  
(b) The $\mathbf{B}$-field found in (a) is far from uniform (it falls sharply with increasing $z$). You can produce a more nearly uniform field by using two such loops a distance $d$ apart as shown in Fig. 4(b). Find the $\mathbf{B}$-field as a function of $z$, and show that $\frac{\partial B}{\partial z} = 0$ at the point midway between them ($z=0$). Now if you pick $d$ just right the second derivative of $B$ will also vanish at the midpoint. This arrangement is known as a Helmholtz coil; it’s a convenient way of producing relatively uniform fields in the lab.

A5.6  
A sphere of linear magnetic material is placed in an otherwise uniform magnetic field $\mathbf{B}_0$. Find the new field inside the sphere.

A5.7  
A perfectly conducting semi-spherical bowl of radius $R$ rotates about the z-axis with angular velocity $\omega$, in a uniform magnetic field $\mathbf{B} = B \mathbf{\hat{z}}$ as shown in Fig. 5.  
(a) Calculate the emf developed between the “south pole” and the equator.  
(b) Which point, a or b shown in the figure, is at the higher potential?
A5.8 A long solenoid with radius R and n turns per unit length carries a time-dependent current I(t) in the \( \hat{\phi} \) direction. Find the electric field \( \mathbf{E} \) at a distance \( s \) from the axis (both inside and outside the solenoid), in the quasistatic approximation.

A5.9 Calculate the magnetic force of attraction between the northern hemispheres of a spinning charged shell.

A5.10 A spherical shell, of radius R, carrying a uniform surface charge \( \sigma \), is set spinning at angular velocity \( \omega \). Find its magnetic dipole moment \( \mathbf{m} \) and the vector potential produced by \( \mathbf{m} \) at a point \( r > R \).

A5.11 Find (a) a complete solution and (b) a spherical harmonic expansion of the vector potential \( \mathbf{A} \) and magnetic induction \( \mathbf{B} \) for a circular current loop of radius R, lying in the x-y plane, centered at the origin and carrying a current \( I \).

A5.12 Prove:
\[
\vec{B} = \frac{\mu_0}{4\pi} \nabla \times \left( \frac{\vec{m} \times \hat{r}}{r^2} \right) = \frac{\mu_0}{4\pi} \frac{1}{r^3} \left[ 3(\vec{m} \cdot \hat{r}) \hat{r} - \vec{m} \right],
\]
where \( \vec{m} = m_r \hat{r} + m_\theta \hat{\theta} + m_\phi \hat{\phi} \) is a constant magnetic dipole moment, \( \vec{r} \neq 0 \).

A5.13 Prove the identity:
\[
\int_{V'} (f \vec{J} \cdot \nabla' g + g \vec{J} \cdot \nabla' f + f \nabla' \cdot \vec{J}) d\tau' = 0,
\]
where \( f(\vec{r}') \), \( g(\vec{r}') \) are well behaved functions of \( r' \). \( \vec{J}(\vec{r}') \) is localized but not necessarily divergenceless.

A5.14 Use the identity in A5.13 to prove:
\[
\int_{V'} \vec{J}(\vec{r}') d\tau' = 0,
\]
where \( \vec{J}(\vec{r}') \) is a localized steady current density, therefore \( \nabla' \cdot \vec{J}(\vec{r}') = 0 \).

A5.15 Use the identity in A5.13 to prove:
\[
\vec{r} \cdot \int_{V'} \vec{J} d\tau' = -\frac{1}{2} \left[ \vec{r} \times \int_{V'} (\vec{r}' \times \vec{J}(\vec{r}') d\tau') \right],
\]
which leads to
\[
\vec{r} \cdot \int_{V'} \vec{r}' \cdot \vec{J}(\vec{r}') d\tau' = -\frac{1}{2} \vec{r} \times \int_{V'} (\vec{r}' \times \vec{J}(\vec{r}') d\tau'),
\]
\( \vec{J}(\vec{r}') \) is a localized steady current density, therefore \( \nabla' \cdot \vec{J}(\vec{r}') = 0 \).

A5.16 Use the identity in A5.13 to prove:
\[
\int_{V'} \vec{r}' \cdot \vec{J}(\vec{r}') d\tau' = 0.
\]
(\( \nabla' \cdot \vec{J}(\vec{r}') = 0 \))

A5.17 For the circuit C shown in Fig. 5.19 (page 210 in Jackson’s book) prove
\[
\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt} \int \vec{B} \cdot d\vec{a} = -\int \frac{\partial B}{\partial t} \cdot d\vec{a} - \oint (\vec{B} \times \vec{v}) \cdot d\vec{l}
\]
Where \( \vec{v} \) is the velocity of the circuit C.
Chapter 6

Jackson: 6.4, 6.8, 6.9, 6.10, 6.11, 6.13, 6.14, 6.20

A6.1 Suppose \( \vec{E}(\vec{r},t) = -\frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} \theta(vt-r)\hat{r} ; \vec{B}(\vec{r},t) = 0 \). Show that these fields satisfy all of Maxwell’s equations, and determine \( \rho \) and \( \vec{J} \). Describe the physical situation that gives rise to these fields.

A6.2 An infinite straight wire carries the current

\[
I(t) = \begin{cases} 
0, & t \leq 0 \\
I_0, & t > 0 
\end{cases}
\]

That is, a constant current \( I_0 \) is turned on abruptly at \( t=0 \). Find the resulting electric and magnetic fields.

A6.3 A piece of wire bent into a loop, as shown in Fig. 1, carries a current that increases linearly with time: \( I(t) = kt \). Calculate the retarded vector potential \( \vec{A} \) at the center. Find the electric field at the center. Why does this (neutral) wire produce an electric field? (Why can’t you determine the magnetic field from this expression for \( \vec{A} \)?)

A6.4 Suppose you take a plastic ring of radius \( a \) and glue charge on it, so that the line charge density is \( \lambda_0 |\sin(\theta/2)| \). Then you spin the loop about its axis at an angular velocity \( \omega \). Find the (exact) scalar and vector potentials at the center of the ring.

A6.5 Determine the force on the “northern” hemisphere of a uniformly charged solid sphere of radius \( R \) and charge \( Q \) as shown in Fig. 2.

A6.6 (a) Consider two equal point charges \( q \), separated by a distance \( 2a \). Construct the plane equidistant from the two charges. By integrating Maxwell’s tensor over this plane, determine the force of one charge on the other.
(b) Do the same for charges that are opposite in sign.

A6.7 Consider an infinite parallel-plate capacitor, with the lower plate (at \( z = -d/2 \)) carrying the charge density \( -\sigma \), and the upper plate (at \( z = +d/2 \)) carrying the charge density \( +\sigma \).
(a) Determine all nine elements of the stress tensor, in the region between the plates. Display your answer as a 3x3 matrix:

\[
\begin{pmatrix}
T_{xx} & T_{xy} & T_{xz} \\
T_{yx} & T_{yy} & T_{yz} \\
T_{zx} & T_{zy} & T_{zz}
\end{pmatrix}
\]
(b) Use equation \( \vec{F} = \int_S \vec{T} \cdot d\vec{a} - \varepsilon_0 \mu_0 \frac{d}{dt} \int_V \vec{S} d\tau \) to determine the force per unit area on the top plate. Compare equation \( \vec{f} = \frac{1}{2\varepsilon_0} \sigma^2 \hat{n} \).

(c) What is the momentum per unit area, per unit time, crossing the xy plane (or any other plane parallel to that one) between the plates?

(d) At the plates this momentum is absorbed, and the plates recoil (unless there is some non-electrical force holding them in position). Find the recoil force per unit area on the top plate, and compare your answer to (b).

A6.8 Imagine a very long solenoid with radius \( R \), \( n \) turns per unit length, and current \( I \). Coaxial with the solenoid are two long cylindrical shells of length \( l \) – one, inside the solenoid at radius \( a \), carries a charge \( +Q \), uniformly distributed over its surface; the other, outside the solenoid at radius \( b \), carries charge \( -Q \) (\( l \gg b \), see figure). When the current in the solenoid is gradually reduced, the cylinders begin to rotate. Where does the angular momentum come from?

Chapter 7

Jackson: 7.1, 7.2, 7.4, 7.12, 7.20, 7.22, 7.23, 7.28, 7.29

A7.1 Write down the (real) electric and magnetic fields for a monochromatic plane wave of amplitude \( E_0 \), frequency \( \omega \), and phase angle zero that is

(a) traveling in the negative x direction and polarized in the z direction;

(b) traveling in the direction from the origin to the point (1,1,1), with polarization parallel to the xz plane. In each case, sketch the wave, and give the explicit Cartesian components of \( \vec{k} \) and \( \hat{n} \).

A7.2 (a) Picture the electron as a uniformly charged spherical shell, with charge \( e \) and radius \( R \), spinning at angular velocity \( \omega \). Calculate the total energy contained in the electromagnetic fields.

(b) Imagine an iron sphere of radius \( R \) that carries a charge \( Q \) and a uniform magnetization \( \vec{M} = M\hat{z} \). The sphere is initially at rest. Compute the angular momentum stored in the electromagnetic fields.

A7.3 (a) Show that the skin depth in a poor conductor (\( \sigma \ll \omega \varepsilon \)) is \( (2/\sigma)\sqrt{\varepsilon/\mu} \) (independent of frequency). Find the skin depth (in meter) for pure water.

(b) Show that the skin depth in a good conductor (\( \sigma \gg \omega \varepsilon \)) is \( \lambda/2\pi \) (where \( \lambda \) is the wavelength in the conductor). Find the skin depth (in nanometers) for a typical metal (\( \sigma \approx 10^7 \text{ (}\Omega\text{m})^{-1} \) in the visible range (\( \omega \approx 10^{15} / \text{s} \), assuming \( \varepsilon \approx \varepsilon_0 \) and \( \mu \approx \mu_0 \). Why are metals opaque?

(c) Show that in a good conductor the magnetic field lags the electric field by 45°, and find the ratio of their magnitudes. For a numerical example, use the “typical metal” in part (b).
A7.4  
(a) Suppose you imbedded some free charges in a piece of glass. About how long would it take for the charge to flow to the surface?
(b) Silver is an excellent conductor, but it’s expensive. Suppose you were designing a microwave experiment to operate at a frequency of $10^{10}$ Hz. How thick would you make the silver coatings?
(c) Find the wavelength and propagation speed in copper for radio waves at 1 MHz. Compare the corresponding values in air (or vacuum).

A7.5  
Calculate the reflection coefficient for light at an air-to-silver interface ($\mu_1 = \mu_0$, $\varepsilon_1 = \varepsilon_0$, $\sigma = 6 \times 10^7 \text{ (}\Omega m\text{)}^{-1}$, at optical frequency ($\omega = 4 \times 10^{15} \text{ / s}$).

A7.6  
(a) Shallow water is non-dispersive; the waves travel at a speed that is proportional to the square root of the depth. In deep water, however, the waves can’t “feel” all the way down to the bottom — they behave as though the depth were proportional to $\lambda$. (Actually, the distinction between “shallow” and “deep” itself depends on the $\lambda$ : if the depth is less than $\lambda$, the water is “shallow”; if it substantially greater than $\lambda$, the water is “deep”.) Show that the wave velocity of deep water waves is twice the group velocity.
(b) In quantum mechanics, a free particle of mass $m$ traveling in the $x$ direction is described by the wave function $\Psi(x,t) = Ae^{i(px-Et)/h}$, where $p$ is the momentum, and $E=p^2/2m$ is the kinetic energy. Calculate the group velocity and the wave velocity. Which one corresponds to the classical speed of the particle? Note that the wave velocity is half the group velocity.

A7.7  
Assuming negligible damping ($\gamma_j = 0$), calculate the group velocity ($v_g = d\omega/dk$) of the waves described by equations $E(z,t) = E_0e^{ikz-\omega t}$ and $k = \omega/c\sqrt{\varepsilon_r}$.

\[ v_g \leq \frac{\omega}{c} \left[ 1 + \frac{Ne^2}{2\varepsilon_0} \sum_j \frac{f_j}{\omega_j^2 - \omega^2 - i\gamma_j\omega} \right]. \]  
Show that $v_g < c$, even when $\omega > c$.

**Chapter 9**


A9.1  
An insulating circular ring (radius $b$) lies in the $xy$ plane, centered at the origin. It carries a linear charge density $\lambda = \lambda_0 \sin \phi$, where $\lambda_0$ is constant and $\sin \phi$ is the usual azimuthal angle. The ring is now spinning at a constant angular velocity $\omega$ about the $z$ axis. Calculate the power radiated.

A9.2  
A particle of mass $m$ and charge $q$ is attached to a spring with force constant $k$, hanging from the ceiling (see figure). Its equilibrium position is a distance $h$ above the floor. It is pulled down a distance $d$ below equilibrium and released, at time $t=0$.

(a) Under the usual assumptions ($d<<\lambda <<h$), calculate the intensity of the radiation hitting the floor, as a function of the distance $R$ from the point directly below $q$. (Note: The intensity
here is the average power per unit area of floor). At what R is the radiation most intense? Neglect the radiative damping of the oscillator.

(b) As a check on your formula, assume the floor is of infinite extent, and calculate the average energy per unit time striking the entire floor. Is it what you'd expect?

(c) Because it is losing energy in the form of radiation, the amplitude of the oscillation will gradually decrease. After what time $\tau$ has the amplitude been reduced to $d/e$? (Assume the fraction of the total energy lost in one cycle is very small).

A9.3 A radio tower rises to height h above flat horizontal ground. At the top is a magnetic dipole antenna, of radius b, with its axis vertical. FM station KRUD broadcasts from this antenna at angular frequency $\omega$, with a total radiated power $P$ (that's average, of course, over a full cycle). Neighbors have complained about problems they attribute to excessive radiation from the tower – interference with their stereo systems, mechanical garage doors opening and closing mysteriously, and a variety of suspicious medical problems. But the city engineer who measured the radiation level at the base of the tower found it to be well below the accepted standard. You have been hired by the neighborhood association to assess the engineer's report.

(a) In terms of the variables given (not all of which may be relevant, of course), find the formula for the intensity of the radiation at ground level, a distance R from the base of the tower. You may assume that $a << c/\omega << h$. [note: we are interested only in the magnitude of the radiation, not in its direction – when measurements are taken the detector will be aimed directly at the antenna].

(b) How far from the base of the tower should the engineer have made the measurements? What is the formula for the intensity at this location?

(c) KRUD's actual power output is 35 kilowatts, its frequency is 90 MHz, the antenna's radius is 6 m, and the height of the tower is 200 m. The city's radio-emission limit is 200 microwatts/cm$^2$. Is KRUD in compliance?
Chapter 11

Jackson: 11.3, 11.4, 11.5, 11.6, 11.8(a), 11.13, 11.16, 11.23

A11.1 Two identical particles of rest mass $m$ are headed toward each other at speed $u$ relative to earth.

a) What is the speed of one particle relative to the other (in terms of $u$ and $c$)?
b) What is the mass does each particle find for the other (in terms of $u$, $c$ and $m$)?
c) If the total energy of one particle relative to earth is $E$, prove that the total energy of one particle relative to the other is $E = \frac{2E^2}{mc^2} - mc^2$. Explain the meaning of this formula.

A11.2 A particle of mass $m$ whose total energy is twice its rest energy collides with an identical particle at rest. If they stick together, what is the mass of the resulting composite particle? What is its velocity?

A11.3 Event A happens at point $(0, 6, 4)$ and at time $t$ given by $ct=2$; event B occurs at $(0,14,9)$ and $ct=12$, both in $S$.

(a) What is the invariant interval between A and B?
(b) Is there an inertial system in which they occur simultaneously? If so, find its velocity (magnitude and direction) relative to S.
(c) Is there an inertial system in which they occur at the same point? If so, find its velocity (magnitude and direction) relative to S.

A11.4 A spaceship is traveling along the line as shown at ordinary speed $u=\frac{\sqrt{3}}{2}c$.

(a) Find the components $u_x$ and $u_y$ of the ordinary velocity.
(b) Find the components $\eta_x$ and $\eta_y$ of the proper velocity.
(c) Find the zeroth component of the 4-velocity, $\eta^0$.

A system $\bar{S}$ is moving in the x-direction with ordinary speed $\frac{3}{4}c$. By using the appropriate transformation laws:

(d) Find the ordinary velocity components $\bar{u}_x$ and $\bar{u}_y$ in $\bar{S}$.
(e) Find the proper velocity components $\bar{\eta}_x$ and $\bar{\eta}_y$ in $\bar{S}$.
(f) Check that $\eta^\mu \eta_\mu = -c^2$.

A11.5 (a) Show that $\bar{E} \cdot \bar{B}$ is relativistically invariant.
(b) Show $F^{\mu\nu} G_{\mu\nu}$ is relativistically invariant

A11.6 A straight wire along the z-axis carries a charge density $\lambda$ traveling in the $+\hat{z}$ direction at speed $v$. Construct the field tensor and the dual tensor at the point $(x, 0, 0)$.
A11.7 A parallel-plate capacitor, at rest in $S_0$ and tilted at a $45^\circ$ angle to the $x_0$ axis, carries charges $\pm \sigma_0$ on the two plates (see figure). System $S$ is moving to the right at speed $v$ relative to $S_0$.

(a) Find $\vec{E}_0$, the field in $S_0$.

(b) Find $\vec{E}$, the field in $S$.

(c) What angle do the plates make with the $x$ axis?

(d) Is the field perpendicular to the plates in $S$?

A11.8 An ideal magnetic dipole moment $\vec{m}$ is located at the origin of an inertial system $\vec{S}$ that moves with speed $v$ in the $x$ direction with respect to inertial system $S$. In $\vec{S}$ the vector potential is $\vec{A} = \frac{\mu_0}{4\pi} \frac{\vec{m} \times \hat{r}}{r^2}$, and the electric potential $V = 0$.

(a) Find the scalar potential $V$ in $S$.

(b) In the non-relativistic limit, show that the scalar potential in $S$ is that of an ideal electric dipole of magnitude $\vec{p} = \frac{\vec{v} \times \vec{m}}{c^2}$ located at $\vec{O}$.