

Test 1  
Calc 1  
9/27/07

Name

(Please print)

### INSTRUCTIONS

- (1) You have 80 minutes to complete this exam.
- (2) There are 11 questions and a total of 100 points.
- (3) Books and notes are not allowed, but you may use a calculator.
- (4) Please circle or box your answer to each question.

Instantaneous velocity when  $t = a$ :

$$\lim_{h \rightarrow 0} \frac{s(a+h) - s(a)}{h}$$

1. (10 pts) Suppose the demand function  $q = D(p)$  for a certain item is linear, where  $p$  is the price per item in dollars and  $q$  is the quantity demanded. If  $p$  increases by \$5, then market research shows that  $q$  will drop by 2 items. Also, 100 items will be demanded if the price is \$550. Find a formula for  $q$  as a linear function of  $p$ .

2. (10 pts) In 1999, the world's population reached 6 billion and was increasing at a rate of 1.3% per year.

(a) Write a formula for the world population,  $P$ , as a function of the number of years  $t$  since 1999.

(b) Use part (a) to estimate the world population in 2020.

3. (10 pts) Solve the equation  $10^{x+3} = 5e^{7-x}$ . Round your answer to 3 decimal places.
4. (5 pts) Find the inverse function  $f^{-1}(t)$  if  $f(t) = 1 + \ln t$ .
5. (10 pts) A cup of coffee contains 100 mg of caffeine, which leaves the body at a *continuous* rate of 17% per hour.
- (a) Write a formula for the amount  $A$  of caffeine in the body  $t$  hours after drinking the coffee.
- (b) Use logarithms to find the half-life of caffeine.

6. (10 pts) The depth of water in a tank oscillates sinusoidally once every 6 hours. If the smallest depth is 5.5 feet and the largest depth is 8.5 feet, find a possible formula for the depth in terms of time (in hours).

7. (10 pts) Suppose the height  $s$  of an object above the ground at time  $t$  is given by  $s = 6 + vt - 16t^2$ , where  $v$  is a constant.

(a) When will the object reach maximum height?

(b) What is the maximum height?

8. (10 pts) State whether the functions below are continuous on the interval  $[-1, 1]$ .

(a)  $f(x) = |x|$

(b)  $g(x) = \frac{|x|}{x}$

(c)  $h(\theta) = \theta \sin \theta$

(d)  $k(t) = \frac{\sin t}{t^2}$

9. (10 pts) Let  $f(x) = \frac{e^{2x-2} - 1}{x - 1}$ .

(a) Make a table of values of  $f(x)$  for  $x = 1.1$ ,  $x = 1.01$ , and  $x = 1.001$ . Round your answers to 4 decimal places.

(b) Make a conjecture about  $\lim_{x \rightarrow 1} f(x)$ .

10. (5 pts) Find a value of  $k$  so that  $\lim_{x \rightarrow 1} \frac{x^2 - kx + 4}{x - 1}$  exists.

11. (10 pts) In a time of  $t$  seconds, a particle is  $s$  meters from a starting point, where  $s = 4t^2 + 3t$ .

(a) Calculate the average velocity between  $t = 1$  and  $t = 3$ .

(b) Use a limit to find the (instantaneous) velocity when  $t = 1$ .

Calc 1  
Test 2  
10/8/07

Name

(Please print)

### INSTRUCTIONS

- (1) You have 80 minutes to complete this exam.
- (2) There are 10 questions and a total of 100 points.
- (3) Books and notes are not allowed, but you may use a calculator.
- (4) Please circle or box your answer to each question.

1. (30 pts) Compute the derivative for each of the following functions:

(a)  $y = 3t^2 + \frac{12}{\sqrt{t}} - \frac{1}{t^2}$

(b)  $y = \pi^x + x^\pi$

(c)  $y = (t^3 - 7t^2 + 1)e^t$

(d)  $y = \frac{t}{t^2 + 1}$

(e)  $y = \sqrt{s^3 + 1}$

(f)  $y = e^{(1+3t)^2}$

2. (5 pts) Sketch the graph of a continuous function  $y = f(x)$  such that  $f'(x) > 0$  for  $x < -2$ ,  $f'(x) < 0$  for  $-2 \leq x \leq 2$ , and  $f'(x) = 0$  for  $x > 2$ .
3. (10 pts) A company's revenue from car sales,  $C$  (in thousands of dollars), is a function of advertising expenditure,  $a$  (in thousands of dollars), so  $C = f(a)$ .
- (a) What does the statement  $f'(100) = 2$  mean in practical terms?
- (b) Suppose the company plans to spend \$100,000 on advertising. If  $f'(100) = 0.75$ , should the company spend more or less on advertising? Explain.
4. (5 pts) Sketch the graph of the height of a particle against time if the velocity is positive and the acceleration is negative.

5. (10 pts) Define a function  $f$  as follows:

$$f(x) = \begin{cases} 0 & x < 0 \\ x^2 & x \geq 0 \end{cases}$$

(a) Sketch the graphs of  $y = f(x)$  and  $y = f'(x)$ .

(b) Is  $f'(x)$  continuous everywhere? Explain.

(c) Is  $f'(x)$  differentiable everywhere? Explain.

6. (5 pts) The height of a sand dune (in cm) is represented by  $f(t) = 700 - 3t^2$ , where  $t$  is measured in years since 2000. Find  $f(5)$  and  $f'(5)$ . Using units, explain what each means in practical terms.

7. (5 pts) Is there a value of  $n$  which makes  $y = x^n$  a solution of the equation  $13x \frac{dy}{dx} = y$ ?  
If so, what value?

8. (10 pts) Find the quadratic polynomial  $g(x) = ax^2 + bx + c$  which best fits the function  $f(x) = e^{2x}$  at  $x = 0$ , in the sense that  $f(0) = g(0)$ ,  $f'(0) = g'(0)$ , and  $f''(0) = g''(0)$ .

9. (10 pts) Suppose  $f$  and  $g$  are differential functions with the values given below:

$x$	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
2	3	2	5	-2

For each of the functions  $h$ , find  $h'(2)$ .

(a)  $h(x) = f(x) + g(x)$

(b)  $h(x) = f(x)g(x)$

(c)  $h(x) = f(g(x))$

10. (10 pts) A particle is moving on the  $x$ -axis, where  $x$  is in cm. Its velocity,  $v$ , in cm/sec, when it is at the point  $x$ , is given by

$$v = x^2 + 3x - 2.$$

Find the acceleration of the particle when it is at the point  $x = 2$ . Give units. (Hint: You need to use the chain rule!)

Calc 1  
Test 3  
11/5/07

Name

(Please print)

### INSTRUCTIONS

- (1) You have 80 minutes to complete this exam.
- (2) There are 11 questions and a total of 100 points.
- (3) Books and notes are not allowed, but you may use a calculator.
- (4) Please circle or box your answer to each question.

1. (10 pts) Find the *exact* value of  $f'(1/3)$  if  $f(x) = \cos(\arctan(3x))$ .
2. (5 pts) If  $g$  is the inverse of  $f$ ,  $f(3) = 4$ ,  $f'(3) = 6$ , and  $f'(4) = 7$ , find  $g'(4)$ .
3. (10 pts) Use implicit differentiation to find the equation of the tangent line to the curve  $\ln(xy) = 2x$  at the point  $(1, e^2)$ . Leave your answer in *exact* form.

4. (10 pts) Find the local linearization of  $f(x) = \frac{2}{x}$  near  $x = 2$ .
5. (5 pts) Find the inflection points of  $f(x) = x^4 + x^3 - 3x^2 + 2$ .
6. (10 pts) Find and classify the critical points of  $f(x) = x^3(1-x)^4$  as local maxima, local minima, or neither.

7. (10 pts) Let  $f(x) = \cos^2 x - \sin x$ .

(a) Find the critical points of  $f$  on the interval  $[0, \pi]$ .

(b) Find the global maximum and minimum values of  $f$  on  $[0, \pi]$ .

8. (10 pts) Suppose the force between two atoms in a molecule is given by  $f(r) = \frac{A}{r^2} + \frac{B}{r^3}$ , where  $A$  and  $B$  are positive constants. What value of  $r$  minimizes the force?

9. (10 pts) Suppose a square-bottomed box, with both top and bottom, has a fixed volume of  $100 \text{ cm}^3$ . What dimensions minimize the surface area? Round your answers to two decimal places.

10. (10 pts) A voltage  $V$  across a resistance  $R$  generates a current  $I = \frac{V}{R}$ . If a constant voltage of 9 volts is put across a resistance that is increasing at a rate of 0.2 ohms per second when the resistance is 5 ohms, at what rate is the current changing? Hint: Use the Chain Rule.

11. (10 pts) Use L'Hopital's rule to compute the following limits:

(a)  $\lim_{x \rightarrow 0} \frac{\sin^2 x}{x}$

(b)  $\lim_{x \rightarrow 1} \frac{\ln x}{x^2 - 1}$

Calc 1  
Test 4  
12/3/07

Name

(Please print)

### INSTRUCTIONS

- (1) You have 80 minutes to complete this exam.
- (2) There are 12 questions and a total of 100 points.
- (3) Books and notes are not allowed, but you may use a calculator.
- (4) Please circle or box your answer to each question.

1. (5 pts) The velocity of a particle moving along the  $x$ -axis is given by  $v(t) = 6 - 2t$  cm/sec. Use a graph to find the exact change in position of the particle from  $t = 0$  to  $t = 4$  seconds.

2. (10 pts) The following table gives the rate (in tons/month) at which pollutants are escaping from a coal works plant:

Time (months)	0	1	2	3	4	5	6
Rate	5	7	8	10	13	16	20

- (a) Give an upper and lower estimate for the total quantity of pollutants that escape in the first 6 months.

- (b) How often would measurements have to be made so that the upper and lower estimates differed by less than one ton during the first six months?

3. (5 pts) What is the exact average value of  $f(x) = \sqrt{1 - x^2}$  over the interval  $-1 \leq x \leq 1$ ? Hint: Sketch a graph.

4. (5 pts) If  $\int_2^5 (3f(x) + 4) dx = 18$ , find  $\int_2^5 f(x) dx$ .

5. (5 pts) Without computing the integral, explain why  $\int_0^2 \sqrt{1+x^3} dx \leq 6$ .

6. (10 pts) A particle moves back and forth along the  $x$ -axis. The velocity  $v$ , in km/hr, as a function of time  $t$  is given below:

$$v(t) = \begin{cases} 10 & 0 \leq t \leq 1 \\ -10 & 1 \leq t \leq 2 \\ 10 & 2 \leq t \leq 3 \end{cases}$$

The particle starts at  $x = 5$ . Graph the distance of the particle from the origin between the times  $t = 0$  to  $t = 3$ , with distance in kilometers and time in hours.

7. (15 pts) Find the indefinite integrals below.

(a)  $\int \frac{4}{t^2} dt$

(b)  $\int \sin x + \cos x dx$

(c)  $\int \left( \frac{y^2 - 1}{y} \right)^2 dy$

8. (10 pts) Find the exact area below the curve  $y = x^3(1 - x)$  and above the  $x$ -axis.

9. (10 pts) Solve the differential equation  $\frac{dy}{dx} = 6x^2 + 4x$ , where  $y(2) = 10$ .
10. (15 pts) A water balloon launched from the roof of a building at time  $t = 0$  has vertical velocity  $v(t) = -32t + 40$  ft/sec at time  $t$  seconds, with  $v > 0$  corresponding to upward motion.
- (a) If the roof of the building is 30 feet above the ground, find an expression for the height  $s(t)$  of the water balloon at time  $t$ .
- (b) What is the average velocity of the balloon between  $t = 1.5$  and  $t = 3$  seconds?
- (c) A 6-foot person is standing on the ground. How fast is the water balloon falling when it strikes the person on the top of the head?

11. (5 pts) Calculate  $\frac{d}{dt} \int_t^\pi \cos(z^3) dz$

12. (5 pts) Write an expression for a function  $f(x)$  such that  $f'(x) = \sin(x^2)$  and  $f(0) = 7$ .