ERRATUM

Corrigendum to the paper

On Bounding the Number of Generators for Fat Point Ideals on the Projective Plane

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There is an error in Theorem 3.1(c) of [1]. In the published version of the paper, the statement is:

Theorem 3.1(c): $\dim_k \ker\mu_t = t - \min\{m, d-m\} + 1$ *if* $\min\{m, d-m\} \le t \le \max\{m, d-m\} - 2$, *(so* $\dim_k \mathrm{cok}\mu_t = \max\{m, d-m\} - t - 1$*).*

A corrected version would say:

Corrected Theorem 3.1(c): *If* $\min\{m, d-m\} \le t \le \max\{m, d-m\} - 2$*, then*

$$\max\{0, 2t - d + 2\} \le \dim_k \ker\mu_t \le t - \min\{m, d-m\} + 1$$

(so $\max\{0, -2t + d - 2\} \le \dim_k \mathrm{cok}\mu_t \le \max\{m, d-m\} - t - 1$*). Moreover, if* $d - m \le m + 1$*, then the upper bounds are equalities.*

The basic problem is the incorrect assumption that $a \le m$, which is made near the end of the proof of Theorem 3.1 (9 lines up from the bottom of page 510). In fact, this inequality can fail, and an example that exhibits the failure is a degree 10 curve with four points of multiplicity 4 and four points of multiplicity 3. Such a curve is smooth and rational on the blow up with $a = 5$ and $m = 4$.

If the claim that $a \le m$ is eliminated, the proof in the paper gives the slightly weaker statement of Theorem 3.1(c) given above. The last three paragraphs of the published proof would be replaced with:

Corrected End of the Proof: We know $a + b = d$. Tensoring the left column of the diagram with $\mathcal{O}_C(t)$ yields:

$$0 \to \mathcal{O}_C(t - (d-m)) \to \mathcal{O}_C(t-a) \oplus \mathcal{O}_C(t-b) \to \mathcal{O}_C(t-m) \to 0, \tag{1}$$

for all t. In particular, if $t = d - m$, then $\Gamma(C, \mathcal{O}_C(t-a) \otimes \mathcal{O}_C(t-b)) \neq 0$, which means $a \le d - m$ and $b\ (= d - a) \ge m$. This gives three possibilities: (i) $a \le d - m \le m \le b$, (ii) $a \le m < d - m \le b$, or (iii) $m < a \le b < d - m$.

For case (i), assume $d - m \le m$. If $t < d - m$, then we see that both $\Gamma(C, \mathcal{O}_C(t - (d-m)))$ and $\Gamma(C, \mathcal{O}_C(t-m))$ are 0, forcing $\Gamma(C, \mathcal{O}_C(t-a) \oplus \mathcal{O}_C(t-b)) = 0$ by (1). Therefore $a \ge d - m$. Since we know $a \le d - m$, we must have $a = d - m$, and consequently $b = m$. Thus

$$\dim_k \ker\mu_t = \begin{cases} 0, & \text{if } t < d - m \\ t - d + m + 1, & \text{if } d - m \le t < m - 1 \\ 2t - d + 2, & \text{if } t \ge m - 1, \end{cases} \tag{2}$$

which is what we wanted.

Now assume $m < d - m$. To handle case (ii), swap m and $d - m$ in the previous argument to see that $a = m$ and $b = d - m$, so

$$\dim_k \ker \mu_t = \begin{cases} 0, & \text{if } t < m \\ t - m + 1, & \text{if } m \leq t < d - m - 1 \\ 2t - d + 2, & \text{if } t \geq d - m - 1. \end{cases} \tag{3}$$

For case (iii), we have $m < a \leq b < d - m$, which leads to

$$\dim_k \ker \mu_t = \begin{cases} 0, & \text{if } t < a \\ t - a + 1, & \text{if } a \leq t < b \\ 2t - d + 2, & \text{if } t \geq b. \end{cases} \tag{4}$$

Parts (a) and (c) of the theorem follow from the extreme cases in (2), (3), and (4); and Part (b) follows from the middle cases of each. ◇

Theorem 3.1(c) was used a few times later in the paper, so its correction forces modifications in of Corollary 3.2, Theorem 3.3(c), and Corollary 3.4, for which corrected statements are given below.

Corrected Corollary 3.2: *Let $\mathcal{C}$ be the sheaf associated to an exceptional curve C on X. Suppose $d = C \cdot L \geq 1$ and $m = \max\{C \cdot E_i \mid 1 \leq i \leq n\}$. Let $u = \min\{d - m, m\}$ and $U = \max\{d - m, m\}$. Then, for $1 \leq r \leq d$,*

(a) *If $0 \leq r < u + 2$, then $\dim R(\mathcal{L} \otimes \mathcal{C}^{\otimes r}|_C) = d - 2r + 2$ and $s(\mathcal{L} \otimes \mathcal{C}^{\otimes r}|_C) = 0$.*

(b) *If $u + 2 \leq r \leq U$, then $\max\{0, d - 2r + 2\} \leq \dim R(\mathcal{L} \otimes \mathcal{C}^{\otimes r}|_C) \leq U - r + 1$ and $\max\{0, 2r - d - 2\} \leq s(\mathcal{L} \otimes \mathcal{C}^{\otimes r}|_C) \leq r - u - 1$, and when $u + 1 \geq d - m$, the upper bounds are equalities.*

(c) *If $r > U$, then $\dim R(\mathcal{L} \otimes \mathcal{C}^{\otimes r}|_C) = 0$ and $s(\mathcal{L} \otimes \mathcal{C}^{\otimes r}|_C) = 2r - d - 2$.*

Corrected Theorem 3.3(c): $\max\{0, U - u - 2\} + (r - U)(r - u - 1) \leq s(\mathcal{L} \otimes \mathcal{C}^{\otimes r}) \leq \binom{U-u}{2} + (r - U)(r - u - 1)$*, for $r > U$.*

Corrected Corollary 3.4: *Let X be the blow-up of $\mathbf{P}^2$ at $n \leq 8$ general points, and let $\mathcal{C}$ be the sheaf corresponding to an exceptional divisor C on X with $d = C \cdot L$, $m = \max\{C \cdot E_i \mid 1 \leq i \leq n\}$, $u = \min\{d - m, m\}$, and $U = \max\{d - m, m\}$. If $1 \leq r \leq d$, then*

$$s(\mathcal{L} \otimes \mathcal{C}^{\otimes r}) = \max\{0, (r - U)(r - u - 1)\}.$$

In all cases, the necessary changes in the proofs are straightforward given the corrected version of Theorem 3.1(c). A complete version of the revised paper, with updated references, is available from http://www.fau.edu/∼sfitchet/.

References

[1] S. Fitchett, On Bounding the Number of Generators for Fat Point Ideals on the Projective Plane, *J. Algebra* 236 (2001), 502–521.