

## Lake Pollution<sup>1</sup>

Environmentalists are concerned about the level of pollution in Lake Woebegone. Much of the pollution flows into the lake from the Istinki River, but there may also be pollution from an aluminum factory located directly on the lake. The current law, passed after heavy lobbying by environmentalists, forces the aluminum factory to close whenever the average concentration of pollutants reaches 16 milligrams per liter of lake water.

The lake has a volume of approximately 4 million liters, and for the purposes of this problem, we will assume the volume stays constant over time. Water flows into the lake from the Istinki River at a rate of approximately 40,000 liters per day, and flows out of the lake, through the Runaway Canal, at the same rate. On October 1st, the lake had 8 milligrams of pollutants per liter of water, and the river consistently has a pollution concentration of 5 milligrams per liter.

1. Assuming that the pollutants are always evenly spread throughout the lake, write a difference equation that models the total mass of pollutants in the lake after  $k$  days. Include an initial condition, and **explain** your model.
2. Write a difference equation that models the *concentration* of pollutants in the lake after  $k$  days. Include an initial condition and explain the relationship between this question and the previous one.

Stop and think about your two models. Both provide essentially the same information, but you may be more comfortable with one than with the other. Think about the relationship between the two and which you would find useful in different situations.

3. Find the mass of pollutants in the lake for each day during October.
4. A ruptured storage tank at the aluminum plant results in 40 kilograms of pollutants being spilled into the lake. Assuming that all of the pollutants reach the lake simultaneously, and immediately mix throughout the lake, determine how long the aluminum factory will be shut down. [Reminder: The lake has a pollution concentration of 8 mg/liter already.]
5. List and discuss at least four factors that would have an impact on the pollution level in the lake, but that were not taken into account by this model.

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<sup>1</sup>Adapted from Interdisciplinary Lively Applications Project (ILAP) C-1 *Lake Pollution*, Part 6, authored by Fox, Goetz, Kolev, Miller, Strickland and King, COMAP, 1995