How Far?

Sabastian and Demeter are planning a summer walking trip around Lake Woeisme in the foothills of the Causcull Mountains. They obtained some information about the area from a US Geological Survey; there's a path (the Square Root route) which skirts the northern edge of the lake and follows the curve $y = 7\sqrt{x}$, and another (the Cubic Camp trail) along the southern edge which follows the curve $y = x^3/49^2$. Here x and y are measured in miles. They will set off from Origin Lodge at the southwest corner of the lake and hike to the waterfall which is at the intersection of the two trails. The goal of this project is to figure out for Sebastian and Demeter which path is shorter.

1. Draw a map of the lake and trails. Label it carefully and compute the distance across the lake from the Lodge to the waterfall.

2. Along each trail there is an overnight shelter for backpackers. Each shelter is located 24.5 miles east of the Lodge and some miles north. Sebastian and Demeter plan to hike to the shelter on the first day and then hike the rest of the way to the waterfall the following day. For each trail, determine the distance as the crow flies from the Lodge to the shelter and from the shelter to the waterfall. Use these to estimate the distance along each trail from the Lodge to the waterfall. Are you estimates less than or greater than the true distances along each trail? Be sure to explain your answers.

3. There are also shelters along each trail located 12.25 miles east of the Lodge and 36.75 miles east of the Lodge. In the case that Sebastian and Demeter take a more leisurely trip, they might spend a night at each of these shelters and do the trip in 4 days instead of 2. Estimate the distance they will have to travel each day. Using these estimates, again estimate the total distance. Are you estimates greater than or less than the actual distances they would travel?

4. Is it possible for Sebastian and Demeter to walk less than 72 miles to get to the waterfall?

5. How could you improve your estimates of the total distances along each trail? Give formulas for your improved estimates (you may want to use Σ notation). Are your improved estimates greater than or less than the actual distances?

6. To make your formulas more familiar looking, factor out Δx. Give an expression for the *exact* distance from the lodge to the waterfall along each trail. Estimate the length of each trail to within half a mile. How do you know that your estimates are accurate enough? Discuss which trail Sebastian and Demeter should take.

Note: The meat of the project is in the last two questions, and grading will weight these questions heavily!